History & Philosophy of Science

History Essay

A Comparison between the reception and scientific application of Euclid's *Elements* in Europe and Asia

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Abstract

The struggle for the status of mathematics takes place in the 16th and 17th century and Euclid's work *The Elements of Geometry* plays a major role.
In this essay I will discuss the remarkable events that took place around that time and I will try to shed some new light on them by comparing them with the events that took place (or noticeably did not) in Asia at the same time. In this essay, I will defend the statement that Euclid's *Elements of Geometry* was not as useful for the Asian scientists as it was to the European scientists.

Introduction

In 1666 Leibniz, then a 20-year old student at the threshold of his career, published his *Dissertatio de arte combinatoria*, in which he wrote that once the mathematical language he had designed would have become common, in case of a misunderstanding it would suffice to take a pencil and a paper and calculate who is right and who is not.

It was at this time that the struggle for mathematics to attain the status of a universal interpreter or even the status of being the fundament of our Universe was decided. The results of this debate would be of fundamental importance to any natural science since that time and to our view of the world.

The struggle I mentioned above takes place in the 16^{th} and 17^{th} century and Euclid's work *The Elements of Geometry* plays a major role.

In this essay I will discuss the remarkable events that took place around that time and I will try to shed some new light on them by comparing them with the events that took place (or noticeably did not) in Asia at the same time. Hopefully this will lead to a further understanding of this matter.

Domain Demarcation

In this essay, I will defend the statement that Euclid's *Elements of Geometry* was not as useful for the Asian scientists as it was to the European scientists. The term 'useful' will require some clarification before I will commence my argumentative structure. With usefulness I mean not usefulness that derives from mere historical interest or the wish for cultural preservation. The term 'useful' I rather introduce meaning the possibility to apply the Euclidian mathematics to their own scientific enterprises under the condition that such an action would be an enrichment or advance. Therefore, 'usefulness' does not require the Euclidian mathematics to be applied to the traditional mathematics (in the European case those two being the same, of course), as long as it is of use to achieving their general goals.

In this essay I will first elaborate on the definition of mathematics (discussing European and Western mathematics as one) I am going to use throughout this text – since we will be dealing with the Asian sciences where mathematics tends to be viewed differently. At the same time I will make some brief remarks about the status of mathematics as a science – what is its realm, what characterises its ways?

Secondly I will shortly introduce Euclid and his writing *The Elements of Geometry.* I will elaborate on the apparent need that was felt in sciences all over Europe in the 17th century – where does this need come from and why was it mathematics they were looking for?

Thirdly I will dwell on the Asian mathematics (before the introduction of Euclid's work) discuss its main characteristics, and then discuss the introduction of Euclid's *Elements* and explain, comparing the reception in the East and the West, why their reception of Euclid shows that it was not of as much use to Asian scientists as it was to the European scientists.

Mathematics and its role in Science

First of all, it seems wise to include a brief discussion of terminology. The first notion that could use some clarification is 'mathematics.' In our Western world mathematics and its domain are clearly demarcated. However, what mathematics is (and what is not) might not be that clear after all when discussing the mathematics of Asia. Therefore here I will clarify my use of the term 'mathematics' and focus on some aspects that are relevant to this discussion. First I will discuss three characteristics of mathematics, namely the necessity of its statements, abstractness and axiomatic reasoning. Additionally, I will quote Kant's philosophy to elaborate on the assumptions that our (Euclidian) mathematical tradition imposes on our worldview.

The first characteristic of mathematics is its abstractness. Mathematics does not concern individual cases, but rather their general aspects. Chris Lorenz points out a similar difference in his book '*De constructie van het verleden. Een inleiding in de theorie van de geschiedenis*.' He writes that a so-called 'covering-law' (which is (1) often used in sciences since the 17th century and (2) is often mathematical in nature – on the latter I will elaborate later) is used to explain events. For example:

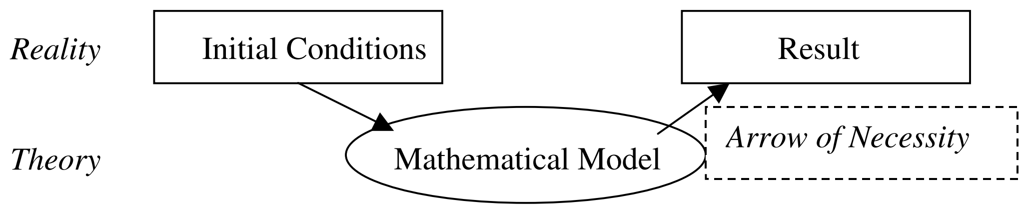
assumption	The theory underlying the *explanans* is correct.
explanans (=what explains (Lat.))	If the temperature is low enough, closed filled plastic bottles will burst open.
Cause	The temperature was low enough.
explanandum (=what is to be explained (Lat.))	These plastic bottles burst open.

The nature of the explanans (the covering-law) is both causal and mathematical. It is formulation include

(1) a number of prerequisites that have to be met (conditions) in order for the law to apply (the causes)
(2) a result

The theory consists the statement that a certain mathematical *model* can be used to describe events that meet certain requirements. The mathematical model itself explains with certainty that if the prerequisites are met, the result will take place (see figure 1). Therefore, a natural scientific theory should be seen apart from the mathematical model it uses. I will come back to this *intermediary role* of mathematics in more detail later.

Figure 1. The epistemological role of mathematics



Mathematics is used in the underlying theory in order to show that in every situation in which the mentioned prerequisites are met, the result must necessarily be the case. The theory would apply a mathematical model of the volume of water and a bottle. As soon as $V_{water} - V_{bottle}$ is larger than a certain constant the conclusion would be that the bottles burst open.
Chris Lorenz writes: “Ten tweede wordt duidelijk dat de oorzakelijke relaties die wetten specificeren, *algemene* relaties zijn, die overal geldig zijn.” (67). In other words, the mathematical covering-law is *abstract*. It does more than just cover one particular incident, rather it explains a abstract generalisation of what happens, i.e. the general aspects of a collection of individual incidents.

Additionally it should be noted that when the prerequisites are met and still the result is not the case, then we would not conclude that our mathematical reasoning is incorrect, but rather that the underlying theory incorrectly applies the mathematical model to this particular (kind of) situation.
Kant elaborates on this matter in his famous *Prolegomena* (23 §2c):

> Zuvörderst muß bemerkt werden: daß eigentliche mathematische Sätze jederzeit Urteile a priori und nicht empirisch sein, weil sie Notwendigkeit bei sich führen, welche aus Erfahrung nicht abgenommen werden kann. Will man mir aber dieses nicht einräumen, wohlan so schränke ich meinen Satz auf die reine Mathematik ein, deren Begriff es schon mit sich bringt, daß sie nicht empirische, sondern bloß reine Erkenntnis a priori enthalte.

Mathematical reasoning logically involves certainty, for it is based on axiom's (based on the Greek axíôma, "worthy") which precisely and uniquely define in which way mathematicians are to perform their reasoning. It is therefore impossible to reject mathematical conclusions that are in line with the protocol-prescribing axioms (e.g. arithmetical rules), unless one rejects its axiom's.

Of especial interest to this essay is a remarkable paraphrasing of these words by Chris Buskes in his preface to "Kennis op het scherpst van de snede, Eindexamencahier Filosofie 2001-2003" (15)

> In zijn *Kritik der reinen Vernunft* stelt Kant dat wij reeds kennis van de empirische werkelijkheid bezitten, zonder dat de empirie geraadpleegd hoeft te worden. Neem bijvoorbeeld de uitspraken: […] 'De kortste verbinding tussen twee punten is een lijn.' "

This statement is quite close to the fourth definition or the first proposition of Book I of Euclid's *Elements*. It is quite remarkable that Buskes claims this statement involves knowledge about the empirical reality, without us having consulted empirical data first.

However, Kant argues (*Prolegomena* 33 §10 emphasis added):

> [S]o bleiben noch Raum und Zeit übrig, […] selbst niemals weggelassen werden können, aber ebendadurch, daß sie reine Anschauungen a priori sind, beweisen, daß sie bloße *Formen unserer Sinnlichkeit* sind, die vor aller empirischen Anschauung, […] der Wahrnehmung wirklicher Gegenstände, vorhergehen müssen.

Kant thinks that space and time and our perception of those precede empirical sense. He writes explicitly in *Kritik der Reinen Vernunft* (76) that they are a form of intuition, i.e. we cannot perceive things without mentally assuming them to be placed in a space and time which we can understand. Now Buskes example of the near-Euclidian sentence is therefore most interesting, since it would imply that the Euclidian space and time (a) seem a natural choice for us (if it were not, then Buskes would not mention them as being 'knowledge about the empirical world.') and (b) are fundamental to empirical reality and not only to how we perceive it.

Finally, John Dee elaborates on the status of mathematics in his Mathematicall Praeface as well. He writes:

> All thinges which are, &haue beyng […] are demed Supernaturall, Naturall, or, of a third being [Thynges Mathematicall]. Things Naturall, are materiall, […] and changeable. Thinges Supernaturall, are, of the minde only, comprehended. […] Thynges Mathematicall […], being middle, [...] are not so absolute and excellent, as thinges supernatural: not yet so base and grosse, as things naturall.

He mentioned before being an admirer of Plato and seems to have adopted his epistemology, placing mathematics in between the Ideas (compatible with Dee's Supernaturall substance) and the Doxa (compatible with Dee's Naturall substance).

In comparison with later philosophers, Dee's view thinks mathematical judgements are not as certain as, for instance, Kant would think they are. Debus writes in his Introduction to the Mathematicall Praeface, that '[f]or some, mathematics really meant iatromathematics, the use of astrology in medicine."

Both views are of especial interest to our discussion later on.

Euclid

Euclid often being referred to as 'Leading mathematics teacher of all time" (O'Connor et al 1), the *Hutchinson Dictionary of Scientific Biography* writes that '[i]t is difficult to see [...] how Euclid's contribution to the science of mathematics could have been more fundamental than it was."

Yet very little is known about Euclid and his life. What is known is that he lived around 300 B.C. and he must have either be a contemporary of Plato or later since the Greek philosopher Proclus refers to Euclid as 'not much younger than these [pupils of Plato]" (O'Connor et al 1) and because there is a distinctly present influence of Plato in Euclid's writing (Hutchinson on *Euclid* par. 2). It is thought to be likely that Euclid attended Plato's Academy, but probably after Plato's death.

Afterwards, he travelled to Alexandria, recently founded at that time, in around 300 B.C. and set up his own school of mathematics there.

Remarkable is J.J. O'Connor's note that perhaps Euclid never existed, that it was a name that a team of mathematicians at Alexandria used, under which to collectively publish their work, in order to remain anonymous and at the same time unify their writings into one consistent final product. After summing up various historical facts that could support this statement, he concludes that we cannot be certain, simply 'having no [more] knowledge about Euclid" (17).

Euclid's *Elements*

Among the works Euclid wrote during his life *The Elements of Geometry* is beyond doubt the most successful one. *The Elements*, for short, is divided into 13 books (Hutchinson on *Euclid* par 4). In these books Euclid apparently sums up all the knowledge that the great Greek mathematicians before him had discovered. O'Connor writes, 'Probably no results in *The Elements* were first proved by Euclid, but the organisation of the material and its exposition are certainly due to him" (22). Among the knowledge brought together are the brilliant discoveries of his famous predecessors Eudoxos and Theaetetos (Veldman 1).

The first six books concern geometry of planes (lines, triangles, squares, parallelograms, circles, etc.) and hypotheses like the famous Pythagorean theorem. Books 7 to 9 concern arithmetic and number theory. Book 10 deals with irrational numbers and book 11 to 13 concern solid geometry (geometry in three-dimensional space) and 'Platonic solids' (rather complicated mathematical shapes such as tetrahedron, icosahedron, dodecahedron) (Hutchinson, par 4).

Table 1. Euclid's Elements

Book	Subject
I – VI	Plane geometry & Hypotheses
VII – IX	Arithmetic & Number Theory
X	Irrational Numbers
XI – XIII	Solid Geometry

Some remarks on Euclid's way of presenting this mathematical material is also in place. The *Hutchinson Dictionary of Scientific Biography* notes that before Euclid two methods of mathematical proof were accepted:

(1) Reasoning from the known to the unknown via logical steps, and
(2) Hypothetically assuming the unknown to be true and reasoning via logical steps backwards to the known.

Euclid uses only the first and therewith set the tone for research to come, for the method became the accepted standard for all mathematical research (and, as is added in *Hutchinson*, scientific research as well).

Euclid's *Elements* has been translated in an astounding number of languages since its creation. Via Arabic it was translated into Latin and from those sources into the European languages, mostly during the 17th century. A detailed overview of the translations based on a collection of articles is inserted as appendix, stating the date of the translation, the name of the person mainly associated with the particular translation and the language. The letter-number codes correspond to the sources this information comes from, and for more detail consult the works cited space.

European Scientific Climate after the Middle Ages

I will now discuss the translations and the scientific background in which they were made in more detail.

During the Middle Ages mathematics had not the same status as the natural sciences. The natural sciences, examining the empirical reality, were generally considered a productive occupation since nature is the work of God, and studying that can be viewed a worthwhile task. Mathematics, however, does not directly involve reality, for example, mathematicians discuss (perfect) circles that tend not to be found in nature. Furthermore, the analytical character of the mathematical research made many people believe that mathematicians merely wasted their time stating the obvious. Therefore, mathematics was granted an intermediate status, being a merely theoretical science.

After the Middle Ages that changed. The many anti-authoritarian reformations in religion together with a broadening horizon (due to new discoveries, for instance Columbus' explorations) brought forth radical changes in science. As authority is questioned previous scientific 'truths' were being re-examined, often more critical than the predecessors.

Of especial interest to our discussion here is the development of perspective in art, which is said to have taken place in the beginning of the 15th century. First of all the actual fact that people were considering how perspective could be included on a painting is a radical change. They were looking for a way to develop their painting so that the drawing did not only make clear what the painter wanted to represent, but that it truly *resembled* what he painted). Second we can note that the need for a mathematical model of this representing of reality on a surface is also evident.

But this change in thinking about representation of reality has also brought about a change in thinking about reality in the first place. Once mathematics had proved successful in guiding painters the way in drawing reality on a canvas, the thought that perhaps nature is in essence mathematical dawned in the 16th- and 17th century mind.

Therefore, in the discussion of mathematics' status, there were

(1) Those who argued that mathematical models are merely a description, and
(2) Those who thought nature is in essence mathematical.

The question that is at stake here is: if a model is correct in describing reality, does that mean we can assume its characteristics or design are inherent to the reality it deals with?

The first group would for instance include Aristotle, who would probably say mathematics alone is insufficient as a way of describing nature, for mathematical equations would only describe *how* a certain object falls, but not *why* it falls or what the essence of falling is. Those were questions Aristotle endeavoured to answer and clearly distinguish his scientific enterprise (together with those of his followers including many medieval scientists) from the new scientific view that arose in the 17^{th} century.

The second group involves for instance Galileo, who wrote in his 1623 published book *Il Saggiatore* (38, emphasis added) that

> [p]hilosophy is written in this grand book - I mean universe - which stands continuously open to our gaze, but which cannot be understood unless one first learns to comprehend the language in which it is written. *It is written in the language of mathematics*, and its characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth

I would like to point out – carefully not to wander outside the domain of this discussion – that this seems a questionable argument at first glance: it is based essentially on the observation that mathematics can be successfully used to describe nature (and one could still argue that that the conclusion that therefore mathematics is not the language of nature cannot be made with logical certainty).

Initially Galileo's 1623 *Il Saggiatore* was well accepted by his contemporaries, even by the Pope, although one must understand that the Pope considered these statements merely hypothetical. Perhaps it was also due to the fact that this Pope (Maffeo Cardinal Barberini (1568-1644)) is believed to have been quite a good friend of Galileo's.
Furthermore, it needs little further explanation that a great need for mathematics is expressed in these words. Galileo calls for a mathematical approach to nature, and for such, a well-founded mathematics is necessary.

Before rounding of this section, I would like to make two additional notes. First of all Euclid's *Elements of Geometry* turned out to be exactly what the European scientists were looking for. Euclidian space (i.e. space in the way Euclid thought about and defined it) is a well defined one, in which place, direction, size and shape can be uniquely and unambiguously represented through numbers. Furthermore, Euclid presented the 17th Century scientists with a sophisticated and well-developed toolbox to use with this Euclidian-space. Now, only the assumption that the reality can truly be understood as a Euclidian space is to be made to allow the scientists to apply this Euclidian toolbox to their observations. However, 20th century scientists such as Einstein would find out that this assumption, that hardly seems worth mentioning, actually is not as valid as it seems to be. I will end the discussion of that particular topic (as it falls outside our domain) with a reference to Gadamer, who wrote in 1960 in *Wahrheit und Methode* that an observer will need to have prior ideas about what he perceives, otherwise he cannot come to an understanding.
Secondly I should add that Galileo was not the only one of that opinion who wrote about it. Debus points out in his *Introduction to the Mathematicall*

Praeface, that Pico della Mirandola (1463-1494), insisted that mathematics was necessary for "further studies in theology" (11).

Mathematics in (East-)Asia

I will now discuss the mathematics that developed in China independently of the European one. I will especially focus on the differences with that of Euclid. Wim Veldman opens his article *Het Chinese Begin* with the sentence "China kent een grote wiskundige traditie."

That tradition noticeably deviates from ours since Euclid's *Elements of Geometry* was not to be introduced to the East-Asiatic region until 1607 (Guangqui and Ricci, refer to Figure 2). The Chinese equivalent of Euclid's *Elements* in terms of fundamentality of influence was Jiuzhang Shuanshu's *The Nine Chapters of Mathematics*, which is thought to have attained its current form around 100 B.C. In 263 it was commentated by Liu Hui and many of its propositions were adopted by mathematicians throughout Eastern-Asia (mainly Japan, Korea and Vietnam).

Table 2. Shuanshu's Nine Chapters

Chapter	Title
I	Rectangular areas
II	Rice
III	Proportional Division
IV	Short width (e.g. determining square root)
V	Constructional Advices
VI	Fair debts

VII	Shortage and Surplus
VIII	Rectangular Configurations (linear equations)
IX	Triangles

Veldman points out that the Chinese mathematics is considerably more practically oriented than ours. He concludes in his article with the remark that studying Chinese mathematics is "een fascinerende ervaring […]" and characterises the mathematics as "meer concreet en meer op de praktijk gericht dan de onze [wiskunde]" (154).

A comparison

Argumentative Structure

Finally we reach the point at which we can draw a comparison between the reception of Euclid in Europe and its reception in Asia. This will construct the main body of my argument, the fundament of which is laid by the information presented before. Its general structure will be as follows. I will first show that if a) a foreign language work is considered valuable, and b) most of those to whom it is of help do not speak the foreign language, it will be translated. Once this is established, I will show that few translations of the work of Euclid were made in the East and those that were being made, differed significantly from the original work.

This will allow us to reach the conclusion as follows:

Argument1 p ==> q
not q, therefore
not p

Secondly I will elaborate on the cultural and language boundary between Europe and Asia that Euclid's work could hardly overcome, no matter how brilliant it might have seemed to us.

First Argumentative Part: Practical use leads to Translation

In order to provide a thorough foundation for my first statement (which is that if a) a foreign language work is considered valuable, and b) most of those to whom it is of help do not speak the foreign language, it will be translated) I will mainly point back to previously presented data. I will show several distinct points that lead up to this conclusion, in summary the latin translation, the uniqueness of our concern over obsolete historical artefacts, Dee's defence of the need for an English translation and the fact that the Romans never translated Euclid.

First of all, I noted before that Euclid was translated into European languages relatively late compared to for instance Arabic (consult the timeline chart figure 2 for verification). This can be due to the fact that before that, Latin was the language generally used by scientists. Such I will not deny, it is not necessary since it is included in the statement. According to Debus, similar translations of Euclid's *Elements* already existed in other European languages, e.g. Italian, German, Spanish and French.

Secondly, I think mere fascination for historical sources is one characteristic that exists only in our modern culture. Debus notes in his *Introduction to the Mathematicall Praeface* (4) that Dee urged Queen Mary to preserve whatever was to be saved of the monastic libraries when those were ordered to be destructed. However, judging from the nature of Dee's Mathematicall Praeface, I would say this action is understandable (the translation of Euclid's *Elements* into English was also a way of preserving knowledge from the past) but it should not be compared to our reasons for preserving historical works. Nowadays, historical cultural heritage is believed to have an intrinsic value, which is why we would not destruct an old temple, even though it might not serve any particular purpose anymore. Judging from for instance the way he sums up all sciences that could benefit from mathematics in his Mathematicall Praeface, convinces that he was not after cultural preservation but practical use.

Thirdly, in his *Mathematicall Praeface*, Iohn Dee briefly touches this matter as well in a rhetorical question (Aiiii^r):

> Besides this, how many a Common Artificer, is there, in these Realmes of England and Ireland, that dealeth with Numbers, Rule, & Cumpasse: Who, with their owne Skill and experience, already had, will be hable (by these good helpes and informations) to finde out, and deuise, new workes, straunge Engines, and Instrumentes: for sundry purposes in the Common Wealth? Or for priuate pleasure? And for the better maintaining of their owne estate?

Without considering the actual content of his words, the fact that he defends the practical use of the work of Euclid to the world of science does indicate that works were not translated without a reason.

Finally, one remarkable historical fact is that Euclid's *Elements of Geometry* was never translated by the Romans. Judging from the many references to its content, it must have been relatively widely spread in the Greek Antiquity. Therefore, the Romans must have had access to at least one copy, but simply were not interested. Archibald argues in his article *The First Translation of Euclid's Elements into English* that 'the first Latin edition of Euclid's *Elements,* […] was made from the Arabic […] around 1120. (445)" I will come back to this issue later in my argumentation.

Concluding we can state that if a) a foreign language work is considered valuable, and b) most of those to whom it is of help do not speak the foreign language, it will most likely be translated.

Once we accept this, we can easily see that the Euclidian mathematical tradition has not been of much use to the Asian mathematicians.
First of all, Mo De and Jiang Zenhua mention in their article *The Recent Chinese and Mongolian Translations of Euclid's Elements* (1) that until their works were published in 1987, only one classical Chinese translation existed from approximately 1607 and no Mongolian one. This is amazing since a) classical Chinese could hardly be read by Chinese who have not devoted themselves to studying those matters, b) classical Chinese is very old and c) it does not relate to modern Chinese like 16th century English relates to modern English. At the same time in Europe, new translations into modern European languages appeared with great rapidity.
Secondly it should be noted that the different Indian translations that Gregg De Young mentioned in his article *Euclidian Geometry in the Mathematical Tradition of Islamic India* deviated enormously. For instance, he reports that Arabic mathematician Nayrizi made a new "apparently heavy" (142) changed edition of al-Hajjaj's translation. He also points out that '[r]ecent studies have […] shown that two distinctly different textual traditions, distinguished in terminology and rearranged orders of definitions and propositions […] exist [within the Ishaq-Thabit translation]" (143). The reader should not be misled to think that the devotion such 'participating translators' shows, is evidence of the usefulness of Euclid's work. Although the devotion must have been real, it shows rather that the Arabic mathematicians tried to extract useful parts of the *Elements*. Now the fact that the translations deviated, therefore apparently the

opinion on what is useful to their ways differed considerably among Indian-Arabic mathematicians-translators. This clearly contrasts with the European tradition in which a) the entire work of Euclid is viewed of use, and with relative consensus and successively b) different translations into the same language were often "word for word identical" (Archibald 446).

Second Argumentative Part: Language and Cultural boundary

For this argument I will present some new data as well as return to previously provided evidence.

First of all, there was an obvious language boundary reported by Martzloff in his article *Note on the Recent Chinese and Mongolian Translations of Euclid's Elements* (201). He writes that the translators had to create neologisms in order to deal with the unclear definitions that were used in Chinese mathematics around the time (which I will discuss in more detail later). Furthermore, the Euclidian style of writing was quite unlike the Chinese one. Martzloff describes the Euclidian presentation as "formal" and says it involves many "repetitions," unlike the Chinese Classical writings, characterised by "extreme conciseness." Therefore, the Chinese mathematician-reader had to deal with an extensive range of neologisms together with an unfamiliar way of writing.

Second, the Chinese were interested much more in the practical application of mathematics, than in the mathematics as a science or even an art – Billingsley uses the term "artes" (ii) to describe mathematics in *The Translator to the Reader*, but it is not exactly the same as "art" in Modern English. The Chinese interest in practical issues is reflected in the choice of issues that were to be touched in *The Nine Chapters*. Veldman also mentioned this in his article (153). In this respect, the Chinese could perhaps be compared with the Romans, who were also more interested in practical application than in theoretical discussions. The reader will agree that Euclid's *Elements* has not been of much use to the Romans.

Thirdly the Chinese mathematics was not axiomatic at all. Veldman writes that "de horzel die met zijn steek de zucht tot axiomatisch redeneren wekt, [in China] weinig slachtoffers [heeft] gemaakt" (153). Martzloff goes even further and demonstrates that the Chinese language does not have words equivalent to the English word "definition" (such equivalents are found throughout European languages). Instead, the Chinese translators of the *Elements* saw themselves forced to make up new words like the compounding of "jie" (limit) and "shuo" (approximately discourse or philosophical doctrine) to represent our notion of "definition." This should be acknowledged to be a major obstruction to scientific application of Euclid's *Elements.* In the 1607 Chinese translations such difficulties were doubtless handled with less care and the result would therefore have been even less useful. But the fact that such notions do not exist must make it very hard for Chinese mathematicians to use Euclidian mathematics. However, Veldman briefly notes in his article that perhaps in Liu Hui's version of the *The Nine Chapters* there originally was a mathematical proof being explained, but the illustration that was refers to is lost.

Finally, the Chinese approach to mathematics is less abstract. They do not use (dimensionless) numbers to represent sizes, such as we would do in mathematics (consider the statement: "take a line of length 1"). In Chapter 5, involving advice on constructions, Liu Hui shows how different geometrical figures relate and for instance how many certain geometrical figures fit into a specified one. Furthermore, while Euclid consistently uses one word for a circle (pointing to an abstract object in the mathematical model representing reality), the Chinese *Nine Chapters* "refer to a circle as a pond, a round field, the base of a silo, and various other context-dependent concepts" (Martzloff, 201) (pointing to a concrete object in reality). Both observations are evidence of a fundamentally different way of thinking. The Chinese mathematicians remained close to the safe shore of considering individual cases than to endeavour in the ocean of the abstract thinking. The non-abstractness of the Chinese mathematics make it hard to reconcile it with works like Euclid's *Elements* and it will therefore not have been of much use to them.

Conclusion

Euclid's *Elements* differs substantially from the mathematical efforts that were made in Asia. Therefore, we can conclude that the Euclidian mathematical work *The Elements of Geometry* was not by far as useful to the Asian mathematicians as it was to the European ones.

We use to think about mathematics as the science which is certain, therefore any mathematical achievement would seem to fit in any other collection of knowledge. Hopefully I have nuanced our view of this 'universally applicable mathematics' in one that perhaps accounts more for influence of culture and does not take the axiomatic assumptions for granted.

Translations of Citations

Chris Lorenz (*De constructie van het verleden*)
"Secondly it appears that the causal relations specified by the covering laws, are *general* relations, and they apply universally."

Immanuel Kant (*Prolegomena*)
"Next this should be noted: truly mathematical sentences are always a priori and not empirical, because they are certain, and perception can never guarantee us such certainty. …"

Chris Buskes (*Kennis op het scherpst van de snede*)
"In his *Kritik der Reinen Vernunft* Kant argues that we have knowledge of the empirical reality without consulting our senses. Consider for instance the statements: […] 'The shortest possible connection between two points is a line.'"

Immanuel Kant (*Prolegomena*)
"Then time and space remain, which have to be present in our perception, but through the fact that they are a priori perceptions, show, that they are merely 'forms of sense,' and must precede […] the empirical perception of any objects."

Wim Veldman (*Het Chinese Begin*)
"China has a long mathematical tradition."
"a fascinating experience."
"more concrete and more practically oriented than our [mathematics]"

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